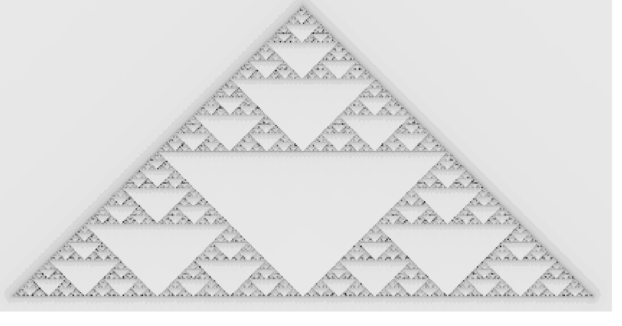
Complexity is around us. Part one: the chaos game

Dawid Lubiszewski

Complex phenomena – like structures or processes – are intriguing scientists around the world. There are many reasons why complexity is a popular topic of research but I am going to describe just three of them. The first one seems to be very simple and says “we live in a complex world”. However complex phenomena not only exist in our environment but also inside us. Probably due to our brain with millions of neurons and many more neuronal connections we are the most complex things in the universe. Therefore by understanding complex phenomena we can better understand ourselves. On the other hand studying complexity can be very interesting because it surprises scientists at least in two ways. The first way is connected with the moment of discovery. When scientists find something new e.g. life-like patterns in John Conway’s famous cellular automata The Game of Life (Gardner 1970) they find it surprising: *In Conway's own words, When we first tracked the R-pentamino... some guy suddenly said "Come over here, there's a piece that's walking!" We came over and found the figure...*(Ilachinski 2001). However the phenomenon of surprise is not only restricted to scientists who study something new. Complexity can be surprising due to its chaotic nature. It is a well known phenomenon, but restricted to a special class of complex systems and it is called the butterfly effect. It happens when changing minor details in the system has major impacts on its behavior (Smith 2007). This means that if we do not have a complete knowledge about the system then our prediction can vary and in some cases this prediction can be totally useless. The last but not least complexity is worth studying because in order to deal with it the scientists create a new language. They talk about fractals, stigmergy, self-organization, emergence and other phenomena. This new language sometimes allows us to describe something very complex in a very simple way and this is also surprising.

Having that all in mind I would like to focus in this introductory article on one example of complexity, that is structural complexity where complex geometrical patterns appear as the result of the simple rules of an algorithm – known as the chaos game. Therefore the aim of this paper is an invitation to more advanced research on complexity.

The chaos game is an algorithm invented in the eighties by British mathematician Michael Barnsley (1988) and has been used to create graphics of fractals and other figures. The simplicity of this method is surprising because in order to create such a beautiful fractal as the Sierpinski triangle (picture 1) the knowledge of higher mathematics or graphic abilities is not important. What you need is a sheet of paper, pencil, dice and some spare time (Peitgen, Jürgens and Saupe 2004). The algorithm to create a Sierpinski triangle is described below.

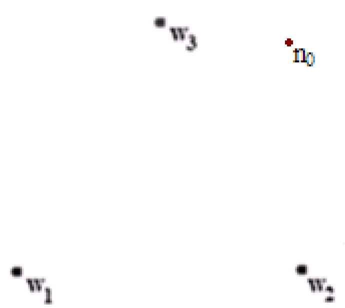


Picture 1: Sierpinski triangle created in Fractal Explorer 2.02.

The first step is to draw three points on the paper, which looks like the apexes of a regular triangle. Each point is named as followed: w_1, w_2 and w_3 (picture 2).

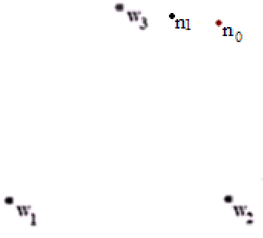
Picture 2: First three points.

In the next step one has to choose the next point n_0 at random and draw it on the paper. It is the “leading” point. In contrast to the first three points, which are stable, this one is going to move but after every move it will leave a trace – a small dot on the paper (picture 3).



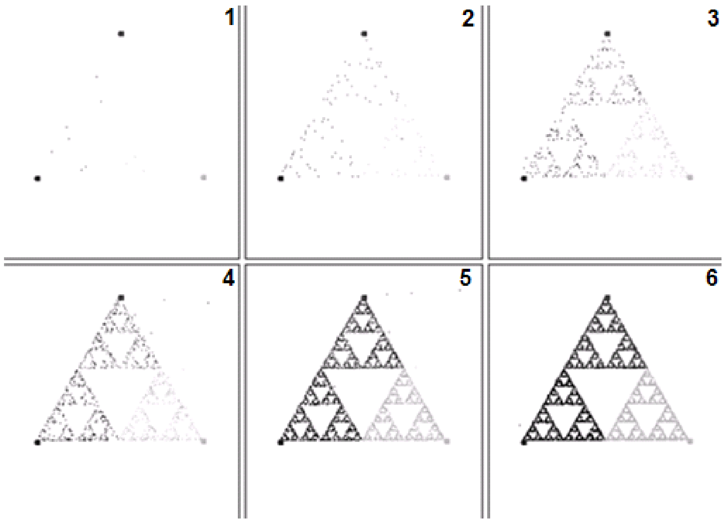
Picture 3: Leading point n_0.

From now at each turn one has to throw a dice and do the following steps. If one tossed 1 or 2 the next leading point (n_1) will be exactly in the midway between n_0 and w_1. In the case of 3 or 4 the next point will be midway between w_2 and n_0. In the last case, that is 5 or 6 the new point will be midway between w_3 and n_0. The last situation is pictured below (picture 4).



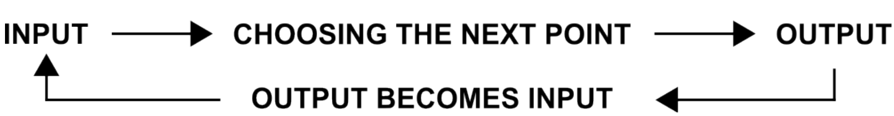
Picture 4: The new leading point (n1) in case of 5 or 6.

After drawing the next point one has to throw a dice again to choose the new point. The whole procedure has to be repeated until an interesting picture appears. Therefore drawing a fractal using this method is not a complicated procedure. However it is very boring and takes a lot of time. Consequently it is much easier to use a computer program randomly choose the next point and project the final picture on the screen. In the next picture (picture 5) are shown the results of a work done by computer program in different time steps (in each time step one leading point is chosen). The first picture shows the points drawn after ten steps, the second after a hundred steps, the third after 500, the next after 1000, the following after 5000 and the last after 10000 time steps. It does not matter where one places the first point (n_0) the final picture is going to be the same - the Sierpinski triangle.



Picture 1: Pictures generated by computer program in different time steps. The program is available on the following page http://www.shodor.org/interactivate/activities/TheChaosGame (date of access 15.10.2010).

The chaos game is not really a game. There are no winners, losers and most of all any players. “Game” is just in the name. The mechanism of this algorithm is very simple: it is the process of continuous iteration. In the other words it repeats the same instruction over time, where the output of one calculation becomes the input for another. That means that feedback plays a crucial role in the chaos game, where one leading point n_1 (input) influences the position of the next leading point n_2 (output), which in turn affects the position of another leading point, that is n_2 becomes a input (picture 6).



Picture 6: Feedback in chaos game.

It has been shown that the chaos game is a very simple method to create fractal images. The picture is drawn as the result of following iterations. Therefore the chaos game is an example of an iterated function system (IFS) which is used to create images e.g. fractals. In mathematical terms the chaos game refers in general to the method of constructing geometric figures. The IFS method discovered by Barnsley has been used in computer science in the area of graphic compression. Barnsley (1988) is famous because one of his IFS algorithms creates an image of fern which looks like the Black Spleenwort (picture 7). The image is a fractal and it has been named after Barnsley so it is called the Barnsley Fern and is very popular not only among scientists.

Picture 2: Barnsley fern generated by IFS in Fractal Explorer 2.02.

The chaos game is not only a useful tool to create impressive computer graphics but also is an intriguing case study in debates about the nature of the world. Are the computer simulations telling us something about the nature? Are they really mimicking the process or just showing one possible way to do it? Does nature use the same short path and similar methods? This leads to another question: is mathematics a language of nature? Or maybe mathematics and computer simulations just show one of the possible descriptions of the universe? Are we really revealing the mysteries of nature or only pointing at possible solutions? This introduction shows one of the possible ways of studying complex phenomena and questions which open the door for further discussion.

Bibliography

1. Barnsley, M. 1988. Fractals everywhere. San Diego: Academic Press, Inc.
2. Gardner, M. 1970. The fantastic combinations of John Conway's new solitaire game 'life'. *Scientifc American* 223: 120-123.
3. Ilachinski, A. 2001. *Cellular Automata. A Discrete Universe.* Singapur: World Scientific: 135.
4. Peitgen, H.-O., Jürgens, H., Saupe, D. 2004. *Chaos and Fractals. New Frontiers of Science. Second Edition.*, Dordrecht: Springer-Verlag New York Inc.: 35.
5. Smith, L.A. 2007. Chaos. A Very Short Introduction, New York: Oxford University Press Inc.: 1-2.